SYNTHESIS OF MICROWAVE FILTERS AND DIRECTIONAL COUPLERS

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E. M. T. Jones* has considered the general synthesis of transmission-line filters consisting of short-circuited, quarter-wave stubs spaced a quarter-wavelength apart on sections of transmission line, each of undetermined characteristic impedance. He has shown that the insertion loss function, P_L, of a symmetrical filter of this type will take the form $P_L = 1 + Q_{n+1}^2(w)/(1+w^2)^n$, where $Q_{n+1}(w)$ is an even or odd polynomial of degree, n + 1, in w with real coefficients, and n is the number of quarter-wavelength series-sections of transmission line. He has also pointed out how equi-ripple performance can be achieved for arbitrary bandwidth and tolerance by means of an ingenious potential analogy suggested by V. H. Grinich. The Grinich transformation is rather involved, with the result that Jones limited his calculations to the coefficients of $Q_{n+1}(w)$ for only a single bandwidth.

Now $w = -\cot \Theta$, where Θ is the common electrical length of the stubs and series line lengths under consideration. If one uses for a frequency variable, $x = \cos \Theta$, instead of w, then it is readily shown that $P_L = 1 + P_{n+1}^2(x)/(1-x^2)$ where $P_{n+1}(x)$ is an even or odd polynomial in x of degree n + 1 with real coefficients. This represents a considerable simplification in the problem, since the degree of the denominator no longer depends on the complexity of the filter. Of course, if $P_{n+1}(x)$ is divisible by $x^2 - 1$, then no shunt elements are required in the realization. The problem of

*E. M. T. Jones, "Synthesis of Wide-Band Microwave Filters To Have Prescribed Insertion Loss," I.R.E. Convention Record, 1956, pp. 119-127.

designing for equal ripple performance reduces then to finching a polynomial, $P_n(x)$, so that $P_n(x)/\sqrt{1-x^2}$ oscillates back and forth between ± 1, a total of n + 1 times in a prescribed interval, $-x_c \leq x \leq x_c$ with $-1 < x_c < 1$. This polynomial, in closed form, is given by,

Theorem: Of all functions of the form $P_n(x)/\sqrt{a^2 - x^2}$ with fixed leading coefficient, $\left\{(a + \sqrt{a^2 - 1})\, T_n(x) - (a - \sqrt{a^2-1})\, T_{n-2}(x)\right\} / 2\sqrt{a^2 - x^2}$ departs the least from zero in the interval between ± 1. Here $T_n(x)$ is the Tchebysheff polynomial of degree n and a is real and greater than one. This function is equi-ripple between ± 1 in this range and tends to $T_n(x)$ as $a \to \infty$. It is a member of a family of equi-ripple functions with a given denominator of the form $\sqrt{Q_m(x)}$, where $Q_m(x)$ has degree less than or equal to the degree of $P_n(x)$, real coefficients and no zeros on the real axis between ± 1. The proof parallels that given by Tchebysheff originally when he considered the same problem for a rational function of x with given denominator.

This solution of the approximation problem in closed form not only permits the convenient determination of the optimum performance but allows the determination of the design parameters of a number of classes of practical microwave devices. These include: parallel-coupled, strip-line filters, filters consisting of shunt resonant elements spaced a quarter-wavelength apart on uniform transmission line (including a large class of broad band T. R. tubes) and broad band stub supports for coaxial line.

The first application to be considered is that of the structure of the type shown in Fig. 1. Here we have a series of shunt elements consisting of inductive irises which have all been tuned, synchronously, to the same frequency by means of the shunt capacities. When they are spaced a quarter-wavelength apart on a uniform transmission line, the present theory is applicable over the pass-band of the filter. In this problem, the additional requirement is placed on the synthesis procedure that all the series line lengths have the same impedance as the generator and load. It is observed that this restriction reduces the original problem to one which is exactly determinant, since the number of unknowns just equals the number of defining equations. It is now, however, no longer possible to prove physical realizability. Experience with the solution of a number of cases indicates, however, that the response is realizable with the required structure in all but the very broad band extremes. The response can also be approximated by two low-pass ladder-networks which appear to bound the exact solution.

For an even number of resonant elements, the ideal equi-ripple response is inconsistent with the physical requirement of mid-band match. This problem can be solved by a suitable linear transformation which is then applied to the four element T. R. tube problem.

An interesting application of the theory can be made to the design of broad band stub supports for coaxial line as shown in Fig. 2. The behavior of P_L in the vicinity of $x = 1$ is a measure of the total shunt admittance of the circuit. Thus $\lim_{x \to 1} 2\, R_{n+1}(x) = Y$ for the single stub support of Fig. 2a, while the same expression equals $2Y + Y^2 Z_2$ for the double stub of Fig. 2b. Clearly then for increasing n, an increasing number of stubs are required if some limit is to be placed on the total admittance to be permitted in a single stub. The two cases of Fig. 2 are analyzed in some detail. Of course, the customary formulas for the single stub are obtained.

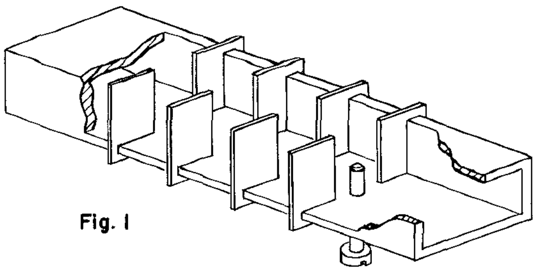

Fig. 1

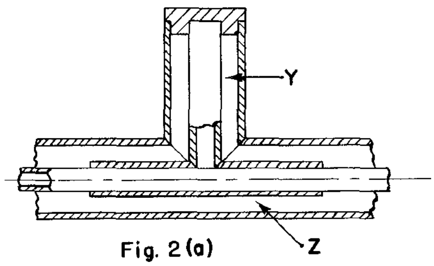

Fig. 2 (a)

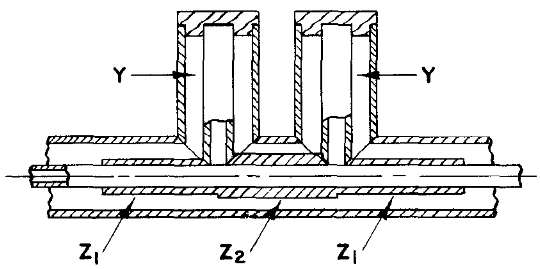

Fig. 2 (b)

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